

A Problem Formulation for Glideslope Tracking in Wind Shear Using Advanced Robust Control Techniques

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Abstract

This paper presents a formulation of the longitudinal glideslope tracking of a transport-class aircraft in severe wind shear and turbulence for application to robust control system design. Mathematical wind shear models are incorporated into the vehicle mathematical model, and wind turbulence is modeled as an input disturbance signal. For this problem formulation, the horizontal and vertical wind shear gradients are treated as real uncertain parameters that vary over an entire wind shear profile. The primary objective of the paper is to examine the formulation of this problem into an appropriate design format for use in μ -synthesis control system design.

1. Introduction

Wind shear is the local variation, at a particular time, of wind velocity with distance, and can be characterized by large rapidly changing variations in wind velocity over short distances. Wind shear can occur from a variety of sources, including convective outflows (such as downbursts associated with thunderstorms and other convective clouds), various types of wind fronts (such as gust fronts, sea-breeze fronts, and air-mass fronts), the jet stream, and terrain-induced wind variability. The magnitude and relative direction of a wind shear are quantified by dividing the velocity difference at two points by the distance between them, and this quantity is referred to as the wind shear gradient. When wind shear occurs at low altitudes in the vicinity of airport runways, it can pose a devastating hazard to aircraft in both the takeoff and landing phases of flight, and various documents are available which report on this flight hazard. One such report is the Federal Aviation Administration report by Shrager (1977), ref. [9], which reported on aircraft accidents or incidents related to low-altitude wind shear occurring from 1964 to 1975. A later report under the auspices of the National Research Council entitled "Low-Altitude Wind Shear and Its Hazard to Aviation" (see Committee ... (1983), ref. [5]) presents details on wind shear arising from all of the above wind shear sources, as well as the hazard each poses to aviation. A conclusion of this report is that the greatest wind shear hazard to aircraft is posed by "downdrafts and outflows produced by convective storms". Thus, a particularly hazardous form of wind shear is the downburst mentioned above, and the microburst, which is a downburst that is small in size (less than 2.5 miles in outflow size) and short in duration (peak winds lasting only 2-5 minutes). The report also states, however, that "serious aircraft accidents have also been caused by terrain-induced and frontal wind shears". Thus downbursts and microbursts are not the only forms of wind shear that are hazardous to aircraft. There are also numerous papers which report on the problem of detecting wind shear, such as a paper by Targ et al. (1991), ref. [11], which presents recent results on a remote wind shear detection sensor under development at the NASA Langley Research Center. This last report cites 26 transport accidents and 3 transport incidents occurring from 1964 to 1985 (for a combined total of 626 fatalities and 236 injuries) that have all been attributed to wind shear. One particularly devastating aircraft landing accident that was attributed to microburst wind shear occurred in 1975 at the John F. Kennedy International Airport and resulted in 112 fatalities. Another devastating encounter with microburst wind shear was a takeoff accident that occurred in 1982 at the New Orleans International Airport and involved 153 fatalities. The most recent catastrophic wind shear related transport accident occurred in 1985 at the Dallas/Fort

Worth Airport. This accident also occurred during landing and resulted in an excess of 100 fatalities.

This paper presents a problem formulation for landing a transport aircraft through severe wind shear and turbulence using advanced robust control techniques. Although the paper by Belcastro & Ostroff (1985), ref. [2], presents an advanced control method for this problem, no specific robustness to wind shear was incorporated into the design. In addition, several ad hoc methods were required in the implementation of the control law during simulation in order to achieve acceptable results. It is postulated that the use of advanced robust control techniques which allow the treatment of wind shear as uncertainties could provide an improved method for designing control systems which are inherently robust to wind shear. However, use of these methods is not yet widespread, and formulating the problem for use with these methods is not commonly understood. This paper therefore focuses exclusively on the details of formulating the problem for use with advanced robust control methods. In particular, the wind shear gradients in the mathematical model of wind shear are treated as real uncertain parameters. The wind profile that will be used herein is based on a reconstruction of the wind profile that occurred during the Kennedy accident mentioned above. The problem will be formulated such that μ -synthesis can be used to design a control law which is robust to an entire wind shear profile. A review of μ -synthesis (and H^∞ theory, as well) is given in Bibel and Stalford (1991), ref. [4].

2. Problem Formulation

The general uncertain system model used in this problem formulation is shown below in Figure 1:

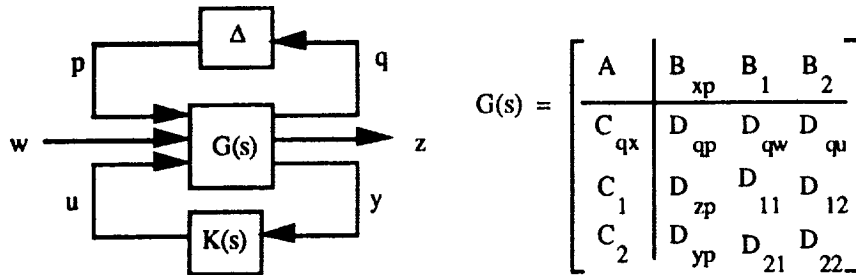


Figure 1. Block Diagram of General Uncertain System

where $G(s)$ is the generalized plant, and $K(s)$ is the controller. The vector w contains all exogenous inputs to the system (including commands, disturbances, and noise), u is the control input vector, z contains the controlled output variables (e.g., tracking errors, control position and rate, etc.), and y represents the measurement vector. The matrix Δ represents a block diagonal matrix of system uncertainties, and the vectors q and p are the inputs and outputs, respectively, to the uncertainty matrix. The nominal $G(s)$ system is represented in state-space form by the matrices A , B_1 , B_2 , C_1 , C_2 , D_{11} , D_{12} , D_{21} , and D_{22} , and the additional B , C , and D matrices in $G(s)$, namely B_{xp} , C_{qx} , D_{qp} , D_{qw} , D_{qu} , D_{zp} , and D_{yp} , represent the interconnection of the uncertainties into the system (see Belcastro, Chang, & Fischl (1991), ref. 4).

The generalized plant, $G(s)$, contains the open-loop plant, $P(s)$, as well as any weighting functions used in the design. The plant, $P(s)$, for this study consists of the aircraft and wind shear dynamic equations. For this problem, $G(s)$ will be formulated as shown in Figure 2:

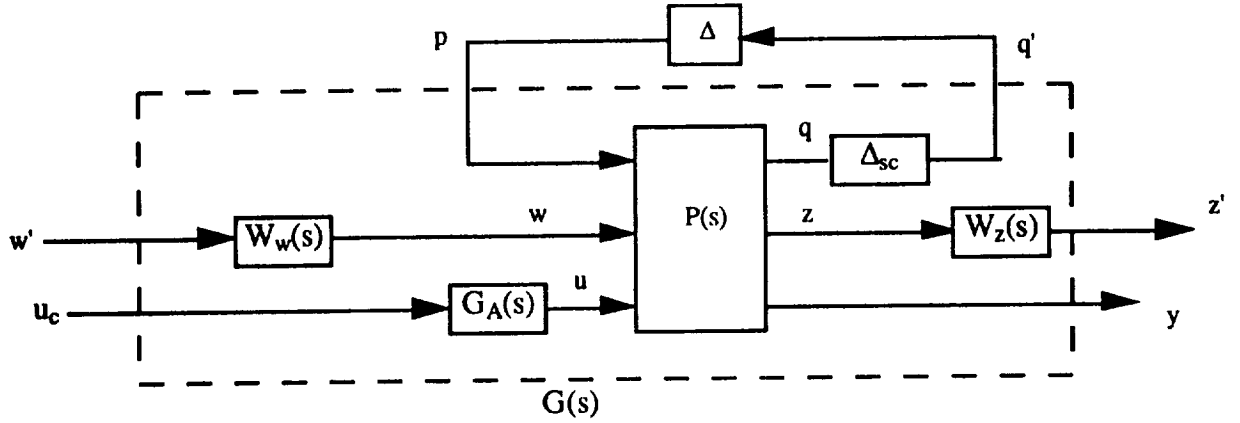


Figure 2. Block Diagram of General Model Subsystems

where $W_w(s)$ represents the exogenous input weighting matrix, Δ_{sc} is a constant scaling matrix associated with the uncertainty model, $W_z(s)$ is the performance weighting matrix, and $G_A(s)$ represents the control actuator model. The overall objective of this study is to track the glideslope, despite severe wind shear and turbulence conditions, during the descent to landing of the aircraft. The subsystem models (depicted in Figure 2) as well as the associated generalized system model (depicted in Figure 1) required to achieve this objective within the framework of this robust control problem formulation are therefore developed in the following subsections.

2.1 Plant Model Formulation

The glideslope tracking problem was formulated for the Transport Systems Research Vehicle (TSRV) B-737 aircraft located at NASA Langley Research Center (see Staff ... (1980), ref. [10]). The linearized mathematical model of the aircraft was based on the following trim conditions:

glideslope:	3 degrees
weight:	80,000 lbs
center of gravity (c.g.):	25% of the MAC (mean aerodynamic chord)
flaps:	40 degrees
airspeed:	125 knots (211.5937 ft/sec)

Plant Model

The state equations for the basic plant model, $P(s)$, represent a linearized perturbation model from the aircraft trim point (defined above) and are given by the following:

$$\dot{x} = Ax + Bu + B_w d_w \quad (1)$$

$$y = Cx + Du + D_w d_w + \eta \quad (2)$$

where: $x = [u \ w \ q \ \theta \ u_w \ w_w \ h]^T \quad (3)$

$$u = [\delta_e \ \delta_T]^T \quad (4)$$

$$d_w = [u_{w_e} \ w_{w_e}]^T \quad (5)$$

$$y = [V_a \ q \ \theta \ h \ \dot{h}]^T \quad (6)$$

$$\eta = [\eta_{V_a} \ \eta_q \ \eta_\theta \ \eta_h \ \eta_{\dot{h}}]^T \quad (7)$$

The first four states in the x -vector (i.e., u , w , q , and θ) represent the four longitudinal aircraft states in the body axes (i.e., longitudinal velocity in the x and z directions of the body axes, pitch rate, and pitch attitude, respectively), u_w and w_w represent the longitudinal and vertical wind velocity components expressed in the positive longitudinal and vertical directions (respectively) of the body axes, and h represents the aircraft altitude along the glideslope. The wind model is comprised of wind velocity components due to both wind shear and turbulence. The wind shear model describes a wind velocity vector (in the inertial Earth-fixed frame of reference) whose horizontal and vertical wind velocity components vary linearly with respect to decreasing altitude along the glideslope. Horizontal and vertical wind shear gradients, U_z and W_z (respectively), were used to represent the linear wind velocity variation relative to the aircraft's descent along the glideslope. The wind shear model was then rotated to the aircraft body axes and expressed as a function of the wind shear gradients U_z and W_z , as well as the associated aircraft states. A full derivation of this model, including the wind shear state equations, is provided in Belcastro & Ostroff (1985), ref. [2]. The wind turbulence model was treated as an input disturbance vector, d_w , whose components consist of the horizontal and vertical wind gust components, u_{wg} and w_{wg} respectively, in the body axes. The Dryden turbulence model was used to provide a description of the turbulence spectrum, and this model is discussed in further detail in Section 2.3.3. The control input vector, u , to the plant consists of elevator deflection (δ_e) and engine thrust (δ_T). The measurement vector, y , consists of aircraft airspeed, V_a , pitch rate, q , pitch attitude, θ , aircraft altitude, h , and altitude rate, \dot{h} , respectively. The vector η represents measurement noise. The matrix elements as well as the trim values and units associated with the vector elements for the above state model are given in the Appendix.

The A and B_w matrices for the above model are of particular interest in this study, because they contain the wind shear gradients, U_z and W_z , which will be treated in this problem formulation as uncertain plant parameters. These matrices are therefore repeated below for convenience:

$$A = \begin{bmatrix} A_{aa} & A_{aw} & 0 \\ A_{wa} & A_{ww} & 0 \\ A_h & 0 & 0 \end{bmatrix} \quad (7 \times 7) \quad (8)$$

$$B_w = \begin{bmatrix} -a_{11} & -a_{12} \\ \vdots & \vdots \\ -a_{71} & -a_{72} \end{bmatrix} \quad (9)$$

The sub-blocks of A are all constant matrices except A_{wa} (2×4), whose elements are linear (affine) functions of the horizontal and vertical wind shear gradients U_z and W_z , respectively, as shown below:

$$\begin{aligned} a_{w11} &= .028944 U_z + .00083845 W_z, & a_{w21} &= -.00083845 U_z + .028944 W_z \\ a_{w12} &= .99916 U_z + .028944 W_z, & a_{w22} &= -.028944 U_z + .99916 W_z \\ a_{w13} &= 0, & a_{w23} &= 0 \\ a_{w14} &= -210.89 U_z - 17.188 W_z, & a_{w24} &= 17.188 U_z - 210.89 W_z \end{aligned} \quad (10)$$

The A and B_w matrices are therefore linear (affine) matrix functions of the horizontal and vertical wind shear gradients U_z and W_z , respectively. The control system is usually designed for the nominal case of no wind shear (i.e., $U_z=0$ and $W_z=0$) so that the design is not "tuned" to a particular wind shear gradient [see Belcastro & Ostroff (1985), ref. 2]. Subsequent to designing the controller, these gradients are typically set to nonzero values for a closed-loop analysis under

various wind shear gradient combinations. For this study, however, the horizontal and vertical wind shear gradients are treated as independent uncertain variables that vary over a defined range, and the range is defined to be representative of an entire wind shear profile during descent. In this way, robustness over an entire wind shear profile can be explicitly incorporated into the design. It should be noted, however, that this approach should not "tune" the design to a particular wind profile since information about which gradient combinations will occur and in what sequence is not provided. Rather, this approach should provide robustness to all gradient combinations within a particular range of values, which are determined from a realistic but severe wind shear profile (such as the Kennedy microburst wind shear profile used in this study). Allowing the gradients to vary independently over all values within the designated range results in a rectangular uncertain parameter space. Intuitively it appears that some subspace (such as a diamond) within this parameter space might be a more accurate representation of the physical microburst wind shear phenomenon. However, further study of microburst wind shear data is required in order to substantiate that this is true in general. In addition, it is unclear whether some preferred subspace exists for realistically representing microbursts as well as other wind shear phenomena, such as terrain-induced and frontal wind shears, and what that subspace might be. Therefore, more extensive study into the structure of these various types of wind shear should be performed in order to define the optimal uncertain parameter subspace for wind shear, and to thereby avoid introducing unnecessary conservatism into the control system design.

Actuator Model

First-order approximations are used to represent the elevator dynamics, throttle dynamics, and engine dynamics. The state model associated with the actuator dynamics is given by the following equations.

$$\dot{x}_A = A_A x_A + B_A u_c \quad (11)$$

$$u = C_A x_A \quad (12)$$

where: $x_A = [\delta_e \delta_{th} \delta_T]^T \quad (13)$

$$u_c = [\delta_{ec} \delta_{thc}]^T \quad (14)$$

and the matrices A_A , B_A , and C_A are defined in the Appendix. The control command inputs contained in the u_c vector are elevator position command (δ_{ec}) and throttle position command (δ_{thc}). These commands will be computed by the control law, $K(s)$.

An uncertainty model for this problem formulation is developed in the next subsection.

2.2 Uncertainty Model Formulation

Uncertainty can be introduced into a general system model as shown above in Figure 1. As stated earlier, Δ represents a block diagonal matrix of system uncertainties, and the additional B , C , and D matrices in $G(s)$ represent the interconnection of the uncertainties into the system (see Belcastro, Chang, & Fischl (1991), ref 4). This uncertainty model will be formulated for the wind shear problem in this subsection. In this study, only uncertainty associated with the wind shear model was considered. In order to obtain the uncertainty model, the uncertain wind shear gradients shown in equation (10) must be separated from the nominal plant system. Details of the uncertainty model development are therefore given next.

2.2.1 Wind Shear Model Uncertainty

As shown in equations (8-10), the A and B_w matrices are linear (affine) matrix functions of the horizontal and vertical wind shear gradients U_z and W_z , respectively. These gradients were considered to be real uncertain variables which vary linearly with additive uncertainty, as follows:

$$U_z = U_{z0} + \delta U_z \quad (15)$$

$$W_z = W_{z0} + \delta W_z \quad (16)$$

where the nominal wind shear gradients, U_{z0} and W_{z0} , were set to zero (since wind shear penetration is not typical of a nominal landing), and the uncertain wind shear parameters, δU_z and δW_z , were allowed to vary over a range defined by the wind shear gradients associated with the 1975 Kennedy landing accident. In order to avoid existence problems in the control system design solution (as a result of setting U_{z0} and W_{z0} to zero), the integrator poles associated with the wind shear model are placed slightly into the left half-plane (e.g., $-.001$ as noted in the Appendix) during the design procedure so that stabilizability requirements for solution are met. The range of gradient values used for the uncertain wind shear parameters, δU_z and δW_z , was determined using a simulation package developed by Stanford Research Institute (SRI) International, which reconstructs the wind profile that occurred during the 1975 Kennedy accident (see Dieudonne (1979), ref. [6]). For this wind shear profile, U_z and W_z were determined in Belcastro & Ostroff (1985), ref. [2], to vary as follows:

$$\begin{aligned} & \text{(Increasing Headwind)} \quad -.25 \leq U_z \leq .39 \quad \text{(Increasing Tailwind)} \\ & \text{(Increasing Updraft)} \quad -.47 \leq W_z \leq .33 \quad \text{(Increasing Downdraft)} \end{aligned}$$

Therefore, for the purposes of this study, the wind shear uncertain parameters were allowed to vary as follows:

$$|\delta U_z| \leq 0.4, \quad |\delta W_z| \leq 0.5$$

where these ranges were selected to cover the wind gradient variation for the Kennedy wind profile described above. Nonsymmetric uncertainty bands, which are more representative of the actual ranges given above, will be included in future work. As described in Belcastro, Chang & Fischl (1991), ref. [3], Morton & McAfoos (1985), ref. [8], and Manning & Banda (1989), ref. [7], the A and B_w matrices can be expanded as follows:

$$A = A_0 + A_\Delta, \quad B_w = B_{w0} + B_{w\Delta} \quad (17)$$

$$\text{and:} \quad M = [A_\Delta \quad B_{w\Delta}] = M_1 \delta U_z + M_2 \delta W_z \quad (18)$$

For this problem formulation, both M_1 and M_2 are rank 2. Thus, as discussed in Morton & McAfoos (1985), ref. [8], and Manning & Banda (1989), ref. [7], the uncertain parameters δU_z and δW_z , will each have to be repeated in the uncertainty block, Δ , of the generalized uncertainty model (Figure 2). As discussed in Belcastro, Chang, & Fischl (1991), ref. [3], the uncertainty interconnection matrices B_{xp} , D_{qw} , C_{qx} , D_{yp} , and D_{qu} matrices can be determined by decomposing the M_i matrices. However, since the uncertain parameters occur only in A and B_w , the matrices B_{xp} , D_{qw} , and C_{qx} are nonzero and D_{yp} , and D_{qu} are zero. The D_{qp} matrix is zero because there are no crossterms of uncertain parameters (e.g., $\delta U_z \delta W_z$) in any of the matrix elements. The B_{xp} , D_{qw} , C_{qx} , D_{yp} , D_{qu} , and D_{qp} matrices associated with this problem are given in the Appendix.

In the generalized uncertainty model, it is desirable to scale the uncertain parameters so that:

$$|\delta' U_z| \leq 1, \quad |\delta' W_z| \leq 1 \quad (19)$$

in order to utilize the standard robust stability and performance μ -test given in Doyle, Wall & Stein (1982). Performing this normalization facilitates the analysis and design process in that the μ

robustness test (without loss of generality) always involves comparing μ to a unity bound, as opposed to recalculating the bound for every problem - or even recalculating the bound within the same problem whenever there is a change in numerical values (such as the uncertainty ranges discussed here). This normalization was therefore accomplished by introducing the following scaling matrix, Δ_{sc} :

$$\Delta_{sc} = \text{diag}[0.4I_2 \ 0.5I_2]^T \quad (4 \times 4) \quad (20)$$

The Δ matrix associated with the uncertainty model is therefore given by:

$$\Delta = \text{diag}[\dot{\delta}_{U_z}I_2 \ \dot{\delta}_{W_z}I_2]^T \quad (4 \times 4) \quad (21)$$

The manner in which Δ_{sc} enters into the model will be shown in the summary of the general system model. The nominal plant model, $P(s)$, consists of the matrices A_o , B_o , C_o , D_o , and D_{wo} , which are also given in the Appendix.

2.2.2 Wind Turbulence Model Uncertainty

For this problem formulation, no uncertainty was modeled for wind turbulence. However, the Dryden turbulence model (described in Dieudonne (1979), ref. [6], and included in the SRI wind simulation package) was used to provide turbulence disturbance weighting. This model will be presented later in Section 2.3.4. As will be discussed there, uncertain parameters associated with this model could be defined and incorporated into the uncertainty model. However, how to do this will be the subject of future work.

2.3 Generalized System Model

The next step in formulating the problem is to define the control and performance objectives, and obtain the associated weighting functions. This is accomplished in the following subsections.

2.3.1 Control Objective

As mentioned previously, the overall objective of this study is to track the glideslope, despite severe wind shear and turbulence conditions, during the descent to landing of the aircraft. The method used in this study to achieve this control objective was to minimize the error in h , V_a , and θ without exceeding actuator position and rate limits. Minimizing the error in h , which represents altitude along the glideslope, provides a means of minimizing deviations from the glideslope during descent, and minimizing errors in V_a and θ provides a means of minimizing the effects of wind shear on the aircraft dynamics. The controlled output vector, z , is therefore given by:

$$z = C_{zx} x + C_{zx_A} x_A + D_{zu} u_c + D_{zc} r_c \quad (22)$$

where:

$$z = [\epsilon_h \ \epsilon_{V_a} \ \epsilon_\theta \ : \ \delta_e \ \delta_{th} \ \dot{\delta}_e \ \dot{\delta}_{th}]^T = [z_c^T \ z_c^T]^T \quad (23)$$

$$r_c = [h_c \ V_{ac} \ \theta_c]^T \quad (24)$$

The first three elements in the controlled vector, z , represent errors associated with altitude, airspeed, and pitch attitude, respectively, and the last four elements represent elevator and throttle positions and rates, respectively. The matrices associated with these controlled variables are given

in the Appendix. The command vector, r_c , represents the commanded altitude, airspeed, and pitch attitude, respectively. For this study, the control effort will focus on tracking a 3-degree glideslope. However, the problem was formulated so that glideslope capture and/or the flare maneuver to transition from the glideslope to touchdown could easily be implemented by incorporating appropriate command models for these variables associated with these maneuvers. For example, a command model for flare might include an exponential flight path command, h_c , with a linearly decreasing airspeed command, V_{ac} , and a linearly increasing pitch attitude command, θ_c . The equations associated with these commands would then be incorporated into the plant state model.

2.3.2 Performance Objectives and Weighting Functions

The performance objectives were formulated relative to the controlled variable vector, z , containing tracking errors and control command variables. The approach used in this study is similar to that described in Balas, Doyle, et al. (1991), ref. [1], and Bibel & Stalford (1991), ref. [4]. The performance requirements associated with glideslope tracking as well as the associated weighting functions are discussed first, followed by the control position and rate limitations of the control actuators along with their associated weights.

The performance requirements associated with glideslope tracking error were specified relative to the following time response characteristics:

Time Constant:	1.0 sec.
Steady-State Error:	0.1%
Step Response Overshoot:	10%

The 1 second time constant corresponds to the natural frequency of the short period mode, and the other values were based on engineering judgement. It should be noted, however, that the design process is iterative, and that the above values are suggested as a reasonable starting point for doing a design for this vehicle. These requirements were then used to define a performance weighting function, W_P , of the following form:

$$W_P(s) = \frac{K_P (s + \omega_1)}{(s + \omega_2)}$$

The time constant requirement was used to establish the weighting function crossover frequency, ω_c , i.e.:

$$\tau = 1.0 \text{ sec} \Rightarrow \omega_c = 1.0 \text{ rad/sec}$$

The steady-state error requirement was used to establish the dc gain of the step response signal (using the final value theorem) as follows:

$$0.1\% \text{ Steady-State Error} \Rightarrow |W_P(s)|_{s \rightarrow 0} = 1/0.001 = 1000$$

where the dc gain is inverted in order to normalize the error response, thereby facilitating the analysis and design process (i.e., μ can be related to unity in the standard μ robustness test). Similarly, the percent overshoot requirement is used to specify transient response performance to a step response (using the initial value theorem):

$$10\% \text{ Overshoot} \Rightarrow |W_P(s)|_{s \rightarrow \infty} = .1$$

This approach is taken from the results presented in Bibel & Stalford (1991), ref. [4]. Using these specifications, $W_P(s)$ was determined as:

$$W_p(s) = \frac{0.1 (s+10)}{(s+.001)} \quad (25)$$

and a frequency response plot is shown in Figure 3:

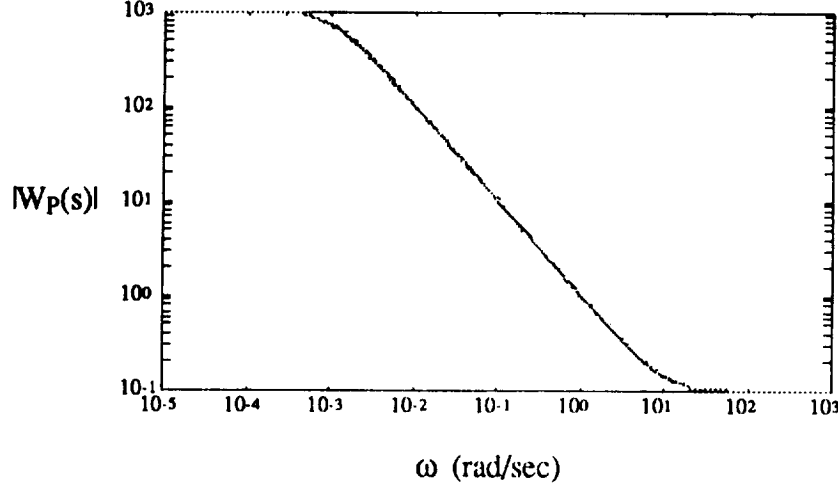


Figure 3. Frequency Response Log Magnitude Plot of the Performance Weighting Function, $W_p(s)$

This performance weighting function is related to the desired sensitivity function and is applied to the three error signals in z . However, if performance requirements for individual error signals vary, the above approach can be used to determine individual weighting functions for each. The state equations for the weighted error variables can then be written as follows:

$$\dot{x}_{wp} = A_{wp} x_{wp} + B_{wp} z_c \quad (26)$$

$$z_c' = C_{wp} x_{wp} + D_{wp} z_c \quad (27)$$

where z_c is defined using equations (22) and (23) and the matrix definitions given in the Appendix as follows:

$$z_c = C_{z_e} x - I_3 r_c \quad (28)$$

and the state vector x_{wp} consists of the states associated with the three error signals in z being passed through the weighting function $W_p(s)$. The matrices A_{wp} , B_{wp} , C_{wp} , and D_{wp} in equations (26) and (27) are defined in the Appendix.

The control position and rate signals can be weighted with constant scalars based on the actuator position and rate limits, as follows:

Elevator Position:	$\delta_c \leq \pm 10 \text{ deg}$	Trim = 3.2 deg	$ \delta_{c\max} = 7 \text{ deg}$
Throttle Position:	$10 \leq \delta_{th} \leq 60 \text{ deg}$	Trim = 17.6 deg	$ \delta_{th\max} = 40 \text{ deg}$
Elevator Rate Limit:		10 deg/sec	
Throttle Rate Limit:		10 deg/sec	

Then the weightings on the control positions and rates were chosen to be:

$$W_{\delta e} = 1/7, \quad W_{\delta th} = 1/40, \quad W_{\delta c(rate)} = 1/10, \quad W_{\delta th(rate)} = 1/10 \quad (29)$$

where the limit values are inverted to normalize the controlled variable outputs - again, to facilitate the analysis and design process.

The overall weighting matrix, $W_z(s)$, is therefore given by:

$$W_z(s) = \text{diag} [W_e \quad W_c] \quad (30)$$

where: $W_e = \text{diag} [W_P(s) \quad W_P(s) \quad W_P(s)] \quad (31)$

$$W_c = \text{diag} [W_{\delta e} \quad W_{\delta th} \quad W_{\delta c(rate)} \quad W_{\delta th(rate)}] \quad (32)$$

Obviously, this weighting matrix is varied during the design process to improve performance. It may even become necessary to vary the performance weighting function on the error signals independently. This weighting strategy is presented as a starting point for the design process.

2.3.3 Disturbance Weighting

The exogenous input weighting matrix, $W_w(s)$, is used to define the frequency spectra associated with the input disturbance vector, d_w , measurement noise vector, η , and command signal vector, r_c . For this problem formulation, $W_w(s)$ was defined as follows:

$$W_w(s) = \text{diag} [W_g(s) \quad I \quad I] \quad (33)$$

where $W_g(s)$ is the weighting matrix associated with the wind gust disturbance vector, d_w , defined in equation (5). The Dryden turbulence model given in Dieudonne (1979), ref. [6], was used to provide lowpass turbulence weighting functions for the longitudinal and vertical wind gust components, u_{wg} and w_{wg} , and this model is repeated below in Figure 4 for convenience:

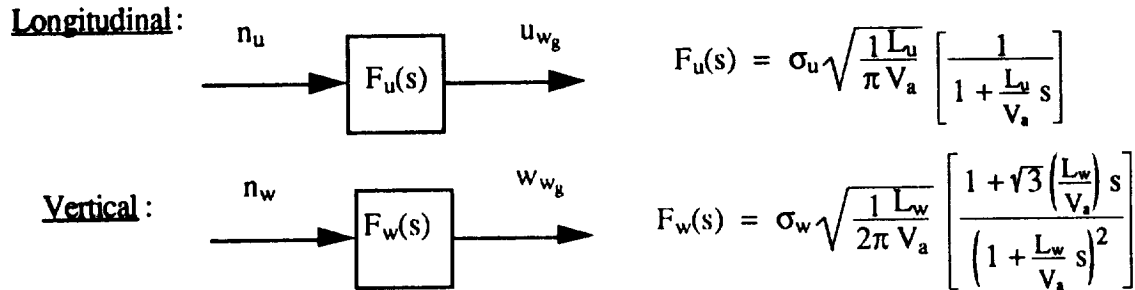


Figure 4. Dryden Turbulence Model

In the above turbulence model n_u and n_w are white, zero mean, unit variance noise signals, V_a represents trim airspeed, and the terms L_u , L_w , σ_u , and σ_w are defined for the Kennedy wind profile as follows in Table 1:

Altitude (ft)	L_u (ft)	L_w (ft)	σ_u (ft/sec)	σ_w (ft/sec)
20.0131	105.7415	10.4003	5.7385	3.9494
100.0328	216.7651	53.0184	6.8355	5.9579
200.0656	306.5945	106.0367	7.4769	7.3419
400.0984	433.5958	212.0407	8.1858	9.0466
600.1640	531.0367	318.0774	8.6246	10.2111
1500.3937	841.1089	795.5052	9.6879	13.4010

Table 1. Turbulence Specifications for the Kennedy Wind Profile

The output signals, u_{wg} and w_{wg} , represent the longitudinal and vertical wind gust components comprising the d_w vector, and these components are given in the body axes with units of ft/sec. The wind disturbance weighting matrix, $W_g(s)$, is therefore determined by the Dryden turbulence model as follows:

$$W_g(s) = \text{diag} [F_u(s) \quad F_w(s)] \quad (34)$$

and the state model associated with this turbulence representation can be formulated as follows:

$$\dot{x}_{wg} = A_{wg} x_{wg} + B_{wg} n_{wg} \quad (35)$$

$$y_{wg} = C_{wg} x_{wg} \quad (36)$$

where:

$$x_{wg} = [u_{wg} \quad w_{wg1} \quad w_{wg2}]^T \quad (37)$$

$$n_{wg} = [n_u \quad n_w]^T \quad (38)$$

$$y_{wg} = [u_{wg} \quad w_{wg}]^T \quad (39)$$

and the matrices A_{wg} , B_{wg} , and C_{wg} are defined in the Appendix. The values from Table 1 that were used in the weighting functions were chosen to yield the highest bandwidth. These values correspond to the first row of the table. A frequency response plot of the longitudinal and vertical transfer functions at these values is shown below in Figure 5:

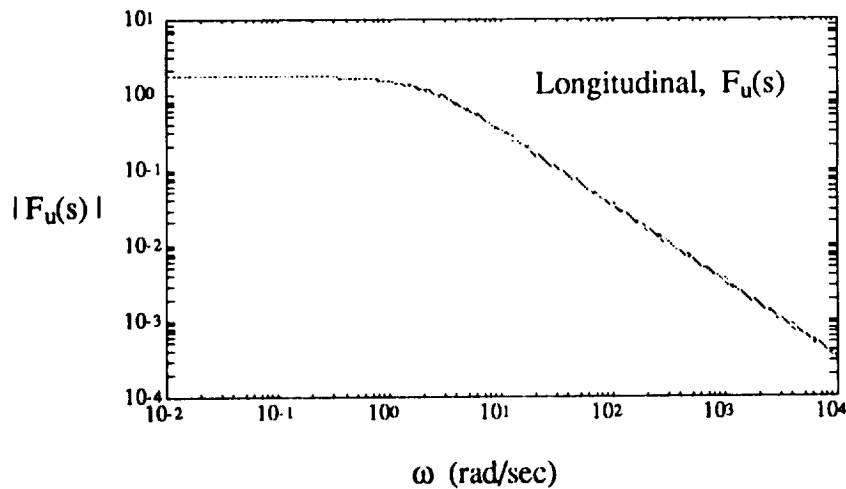


Figure 5a. Frequency Response Plot of the Dryden Turbulence Model Using the Data Given in Row 1 of Table 1

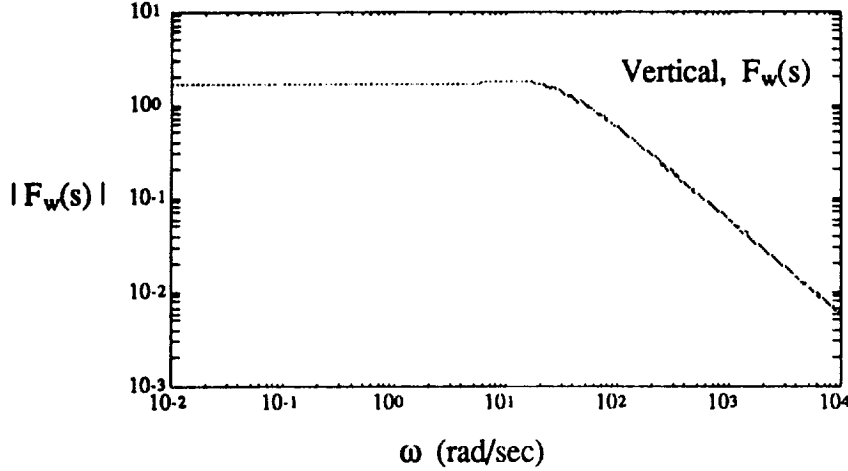


Figure 5b. Frequency Response Plot of the Dryden Turbulence Model Using the Data Given in Row 1 of Table 1

As mentioned previously, the Dryden model could also be used to develop an uncertainty model associated with wind turbulence, where the terms L_u , L_w , σ_u , and σ_w are considered to be real uncertain parameters which vary over the values defined in Table 1 for the Kennedy wind profile. As can be seen by the above transfer functions as well as the A_{wg} , B_{wg} , and C_{wg} matrices given in the Appendix, these parameters enter into the model in a fairly complicated manner. Thus, modeling these parameters as uncertainties will be addressed in future work.

As indicated in equation (33), no dynamic weighting was defined for the measurement noise vector, η , or the command signal vector, r_c . However, measurement noise weighting could be accounted for in the design process by defining an appropriate high-frequency noise spectrum for each sensor measurement signal and incorporating the associated states into the generalized model as demonstrated above for the turbulence model. Similarly, command signal prefiltering could be accounted for in the design model by defining appropriate weighting functions for the command signals in r_c .

2.3.4 Summary of the Generalized System Design Model

The generalized system model as depicted in Figure 1 is now summarized. The vector states, inputs, and outputs for this problem are given as follows:

$$x_G = [x \ x_A \ x_{wg} \ x_{wp}]^T \quad (40)$$

$$p = [p_{u_{z1}} \ p_{u_{z2}} \ p_{w_{z1}} \ p_{w_{z2}}]^T \quad (41)$$

$$w = [d_w \ \eta \ r_c]^T \quad (42)$$

$$q = [q_{u_{z1}} \ q_{u_{z2}} \ q_{w_{z1}} \ q_{w_{z2}}]^T \quad (43)$$

The vectors x , x_A , and x_{wg} are defined in equations (3), (13), and (37), respectively, and the x_{wp} vector represents the three states associated with the performance weighting functions for the three error signals in the control vector, z . The vectors u , y and z of Figure 1 are defined in equations (4), (6), and (23), respectively. The elements of d_w , η , and r_c are defined in (5), (7), and (24), respectively. The generalized plant model, $G(s)$, contains the nominal plant model as well as all weighting and scaling matrices depicted in Figure 2 and used in the design and analysis. This model is summarized for the configuration of Figures 1 and 2 as follows:

$$G(s) = \begin{bmatrix} A_G & B_G \\ C_G & D_G \end{bmatrix} \quad (44)$$

$$A_G = \begin{bmatrix} A_o & B_o C_A & B_{w_o} C_{w_g} & 0 \\ 0 & A_\Delta & 0 & 0 \\ 0 & 0 & A_{w_g} & 0 \\ B_{w_p} C_{z_e} & 0 & 0 & A_{w_p} \end{bmatrix} \quad (45)$$

$$B_G = \begin{bmatrix} B_{xp} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ B_{w_g} & 0 & 0 \\ 0 & 0 & -B_{w_p} \end{bmatrix} & 0 \\ 0 & & B_A \\ 0 & & 0 \\ 0 & & 0 \end{bmatrix} \quad (46)$$

$$C_G = \begin{bmatrix} \Delta_{sc} C_{qx} & \Delta_{sc} D_{qu} C_A & \Delta_{sc} D_{qw} C_{w_g} & 0 \\ \begin{bmatrix} D_{w_p} C_{z_e} & 0 & 0 & C_{w_p} \\ 0 & W_c C_{z_u} & 0 & 0 \end{bmatrix} \\ C_o & D_o C_A & D_{w_o} C_{w_g} & 0 \end{bmatrix} \quad (47)$$

$$D_G = \begin{bmatrix} \Delta_{sc} D_{qp} & \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} & 0 \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} -D_{w_p} \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ W_c D_{z_{uu}} \end{bmatrix} \\ D_{yp} & \begin{bmatrix} 0 & I & 0 \end{bmatrix} & 0 \end{bmatrix} \quad (48)$$

The matrices A_o , B_o , B_{w_o} , C_o , D_o , and D_{w_o} are associated with the nominal plant model, and the matrices B_{xp} , C_{qx} , D_{qw} , D_{qu} , D_{yp} , and D_{qp} are associated with the interconnection of the uncertainty matrix, Δ . The matrices C_{z_e} , C_{z_u} , and $D_{z_{uu}}$ are associated with the controlled variable model, and the matrices A_{w_p} , B_{w_p} , C_{w_p} , and D_{w_p} are associated with the performance weighting matrix, W_e , associated with the performance weighting function $W_p(s)$. All of these matrices are defined in the Appendix. The uncertain parameter scaling matrix, Δ_{sc} , is given in equation (20), and the control weighting matrix, W_c , is defined by equations (32) and (29). The matrix for this problem is defined in equation (21). The only other component in the generalized model of Figure 1 is the controller, $K(s)$, which could now be designed using μ -synthesis. The controller design for this problem formulation will be the subject of future work.

3. Conclusions

This paper has presented a formulation of a glideslope tracking problem through severe wind shear and gusts for application to robust control system design using μ -synthesis. The uncertainty in this problem is associated with the wind shear model and appears as real parameter variations of the horizontal and vertical wind shear gradients. The range of parameter variation was based on the gradient extremes determined from a reconstruction of the wind profile that occurred in the 1975 landing accident at Kennedy International Airport. Formulating this problem for μ -synthesis should provide a means of incorporating robustness to wind shear explicitly into the design process without "tuning" the design to a particular wind gradient or profile. Wind gusts were also included in the model as disturbance inputs to the plant. The Dryden turbulence model was used to provide weighting information. However, this model could also be used to formulate additional wind model uncertainty by treating the turbulence parameters L_u , L_w , σ_u , and σ_w as real uncertain parameters. Future work in the area of wind shear and gust uncertainty modeling will address the inclusion of these turbulence terms as part of the uncertainty model, as well as the determination of an appropriate wind shear parameter subspace. Nonsymmetric uncertainty bands associated with the uncertain parameters will also be addressed. Performance weighting functions were also derived for inclusion in the problem formulation developed in this paper. This problem formulation can be used to obtain a μ -synthesis control system design, although modifications to the design parameters (e.g., the weighting functions) may be required to obtain a final design. The results of the control system design as well as a closed-loop evaluation using a full nonlinear simulation of the aircraft will be presented in a subsequent paper.

Appendix: Model Definition

Plant State Model

$$A = \begin{bmatrix} A_{aa} & A_{aw} & 0 \\ A_{wa} & A_{ww} & 0 \\ A_h & 0 & 0 \end{bmatrix} \quad (7 \times 7)$$

where:

$$A_{aa} = \begin{bmatrix} -.41790E-01 & .98126E-01 & -.49565E+01 & -.32155E+02 \\ -.29344E+00 & -.76071E+00 & .21178E+03 & .96749E+00 \\ -.30862E-03 & -.49839E-02 & -.52858E+00 & -.37077E-03 \\ .00000E+00 & .00000E+00 & .10000E+01 & .00000E+00 \end{bmatrix}$$

$$A_{aw} = \begin{bmatrix} .41790E-01 & -.98126E-01 \\ .29344E+00 & .76071E+00 \\ .30862E-03 & .49839E-02 \\ .00000E+00 & .00000E+00 \end{bmatrix}, \quad A_{wa} = \begin{bmatrix} a_{w11} & a_{w12} & a_{w13} & a_{w14} \\ a_{w21} & a_{w22} & a_{w23} & a_{w24} \end{bmatrix}$$

$$a_{w11} = .028944 U_z + .00083845 W_z, \quad a_{w21} = -.00083845 U_z + .028944 W_z$$

$$a_{w12} = .99916 U_z + .028944 W_z, \quad a_{w22} = -.028944 U_z + .99916 W_z$$

$$a_{w13} = 0, \quad a_{w23} = 0$$

$$a_{w14} = -210.89 U_z - 17.188 W_z, \quad a_{w24} = 17.188 U_z - 210.89 W_z$$

U_z = Horizontal Wind Shear Gradient (Uncertain Parameter)

W_z = Vertical Wind Shear Gradient (Uncertain Parameter)

$$A_{ww} = \begin{bmatrix} a_{w15} & 0 \\ 0 & a_{w26} \end{bmatrix}$$

$$a_{w15} = 0 \quad (a_{w15} = -.001 \text{ for Design Model})$$

$$a_{w16} = 0 \quad (a_{w16} = -.001 \text{ for Design Model})$$

$$A_h = \begin{bmatrix} -.028956 & -.99958 & 0 & 211.3 \end{bmatrix}$$

$$B = \begin{bmatrix} B_a \\ 0 \end{bmatrix} \quad (7 \times 2), \quad B_a = \begin{bmatrix} .40382E-02 & .40218E-03 \\ -.17254E+00 & -.20149E-06 \\ -.21025E-01 & .62988E-05 \\ .00000E+00 & .00000E+00 \end{bmatrix}$$

$$B_w = \begin{bmatrix} -a_{11} & -a_{12} \\ \vdots & \vdots \\ -a_{71} & -a_{72} \end{bmatrix}$$

$$C = \begin{bmatrix} .9997 & .0234 & 0 & 0 & -.9997 & -.0234 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -.028956 & -.99958 & 0 & 211.3 & 0 & 0 & 0 \end{bmatrix}$$

$$D = [0] \text{ (5x2) }, D_w = [0] \text{ (5x2)}$$

Plant Trim Values:

Variable	Trim Value	Units
u	211.53575	ft/sec
w	4.9508090	ft/sec
q	0	rad/sec
θ	-.028960023	rad
u_w	0	ft/sec
w_w	0	ft/sec
h	200	ft
δ_c	3.178	deg
δ_{th}	17.576	deg
δ_T	8,112.717	lbs
V_a	211.5937	ft/sec
U_z	0	ft/sec/ft
W_z	0	ft/sec/ft

Actuator State Model

$$A_A = \begin{bmatrix} 0 & 0 \\ -\omega_{\delta_c} & -\omega_{\delta_{th}} \\ 0 & K_{eng}\omega_{\delta_{eng}} \\ 0 & -\omega_{\delta_{eng}} \end{bmatrix}, B_A = \begin{bmatrix} \omega_{\delta_c} & 0 \\ 0 & \omega_{\delta_{th}} \\ 0 & 0 \end{bmatrix}$$

where: $\omega_{\delta_c} = 16 \text{ rad/sec}$, $\omega_{\delta_{th}} = 10 \text{ rad/sec}$, $\omega_{\delta_{eng}} = 0.5 \text{ rad/sec}$, $K_{eng} = 596 \text{ lbs/deg}$

$$C_A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Uncertainty Model

Nominal Plant Model:

$$A_0 = A \text{ (} A_{wa} = 0 \text{)}, B_0 = B, C_0 = C, D_0 = D, D_{w0} = D_w$$

Uncertainty Matrices:

$$B_{xp} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} (7 \times 4), \quad D_{qw} = \begin{bmatrix} -.028944 & -.99916 \\ .00083845 & .028944 \\ -.00083845 & -.028944 \\ -.028944 & -.99916 \end{bmatrix}$$

$$C_{qx} = \begin{bmatrix} .028944 & .99916 & 0 & -210.89 & 0 & \dots & 0 \\ -.00083845 & -.028944 & 0 & 17.188 & 0 & \dots & 0 \\ .00083845 & .028944 & 0 & -17.188 & 0 & \dots & 0 \\ .028944 & .99916 & 0 & -210.89 & 0 & \dots & 0 \end{bmatrix} (4 \times 7)$$

$$D_{yp} = [0] (5 \times 4), \quad D_{qu} = [0] (4 \times 2), \quad D_{qp} = [0] (4 \times 4)$$

Controlled Variable Model

$$C_z = [C_{z_x} \quad C_{z_u}] = \begin{bmatrix} C_{z_e} & 0 \\ 0 & C_{z_u} \end{bmatrix}$$

$$C_{z_e} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ .9997 & .0234 & 0 & 0 & -.9997 & -.0234 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad C_{z_u} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\omega_{\delta_e} & 0 & 0 \\ 0 & -\omega_{\delta_{th}} & 0 \end{bmatrix}$$

$$D_{z_u} = \begin{bmatrix} 0 \\ D_{z_{uu}} \end{bmatrix} (7 \times 2), \quad D_{z_{uu}} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \omega_{\delta_e} & 0 \\ 0 & \omega_{\delta_e} \end{bmatrix}$$

$$D_{z_e} = \begin{bmatrix} -I_3 \\ 0 \end{bmatrix} (7 \times 3)$$

Performance Weighting Model

$$A_{w_p} = \begin{bmatrix} -.001 & 0 & 0 \\ 0 & -.001 & 0 \\ 0 & 0 & -.001 \end{bmatrix}, \quad B_{w_p} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

$$C_{w_p} = \begin{bmatrix} 9.999 & 0 & 0 \\ 0 & 9.999 & 0 \\ 0 & 0 & 9.999 \end{bmatrix}, \quad D_{w_p} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

Wind Turbulence Model

$$A_{w_g} = \begin{bmatrix} -\frac{V_a}{L_u} & 0 & 0 \\ 0 & -\frac{V_a}{L_w} & 0 \\ 0 & \sigma_w \frac{1}{\sqrt{L_w}} \sqrt{\frac{3V_a}{2\pi}} & -\frac{V_a}{L_w} \end{bmatrix}$$

$$B_{w_g} = \begin{bmatrix} \sigma_u \frac{1}{\sqrt{L_u}} \sqrt{\frac{V_a}{\pi}} & 0 \\ 0 & -V_a \frac{1}{L_w} \left(1 - \frac{1}{\sqrt{3}}\right) \\ 0 & \sigma_w \frac{1}{\sqrt{L_w}} \sqrt{\frac{3V_a}{2\pi}} \end{bmatrix}$$

$$C_{w_g} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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